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DIRECTORATE OF
INTELLIGENCE

Intelligence Report

Investment And Growth In The USSR

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CENTRAL INTELLIGENCE AGENCY
Directorate of Intelligence
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INTELLIGENCE REPORT

Investment And Growth In The USSR

Introduction

One of the principal features of Soviet economic development has been the government's policy of investing the maximum possible amount of the national product. This report explores the possibility that this traditional investment policy is no longer capable of providing the rate of economic growth desired by the Soviet leadership. After World War II, this policy for a time met with much the same sort of success in promoting high rates of economic growth as it had before the war. In the process, however, the investment rate (investment in buildings and equipment expressed as a share of gross national product) increased from 12% in 1950 to 23% in 1960. Since 1960, it has grown more slowly -- to about 26% in 1969.

The steady rise in the investment rate during the 1950s brought about a very rapid increase in the stock of capital in the economy. At the same time, output grew almost as rapidly, so the ratio of capital to output remained at a fairly low level. According to Simon Kuznets, a leading student of comparative economic development, "... the distinctive feature of the USSR record is that so much capital formation was possible without an increase in the capital-output ratio to uneconomically high levels."* He was referring to growth prior to 1958. The USSR now seems to have lost that distinction.

* *Economic Trends in the Soviet Union, Ed. A. Bergson and Simon Kuznets, 1963, p. 357.*

Note: This report was produced solely by CIA. It was prepared by the Office of Economic Research.

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In the 1960s the growth of output of industry, construction, and national income, as announced by the Soviet government, slowed dramatically. The growth in capital stock also declined, but not as much as the growth of output. The resulting fall in the ratio of output to capital was noted by Soviet politicians and technicians alike. Such a decline in the return on capital investment threatened the basic Soviet strategy of economic development. The economic difficulties of this period contributed to Khrushchev's fall from power in 1964 and led to the promulgation of Kosygin's reforms in 1965. At first, Khrushchev's successors tended to treat the decline in the output/capital ratio as a temporary phenomenon resulting from Khrushchev's bad management. More recently, they have reluctantly recognized that a turning point has been reached in the method of achieving economic growth.*

The role of investment and capital in Soviet economic growth is explored in this report by means of an aggregate production function. A production function is a relation between inputs -- usually capital and labor -- and the resulting output, or production. Production functions of one kind or another are often used for medium-range economic forecasting, but in previous work

* *The gist of the leadership's remarks to the December (1969) plenary meeting of the CPSU Central Committee has been reported as follows: "The definite reasons for our difficulties are essentially connected with the fact that we have entered a stage of development that no longer permits us to work in the old manner but demands new methods and new solutions The raising of the effectiveness of social production has indeed become the key problem, primarily because the main factors in our economic growth have changed. If we were previously able to develop the national economy primarily by quantitative factors, i.e., by increasing the number of workers and by high rates of accumulation of capital investments, then henceforth we must count primarily on qualitative factors of economic growth, on raising the effectiveness, the intensification of the national economy." (Pravda, 13 January 1970, p. 1.)*

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on the USSR both the general form and the precise characteristics of the relationship between output and inputs have been usually assumed or specified by analogy with Western practice.

In this report a relatively new form of production function is fitted statistically to the Soviet postwar experience. This function -- known as the Arrow-Chenery-Minhas-Solow function after some of the economists who first proposed it -- has the characteristic of allowing for rapidly diminishing returns to capital. This function is compared with production functions previously used for forecasting Soviet economic growth. The various functions are then used as a basis for discussion of the following questions:

- a. What return on investment can be expected in the USSR in the coming years?
- b. Can the USSR rely on an upswing in the growth of investment -- perhaps at the expense of military expenditures -- to restore the rates of economic growth achieved in the 1950s (or mid-1960s)?

The production functions in this report are based on the past performance of the Soviet economic system -- in particular, on the past efficiency of its economic organization and on the past rate of adoption of new technology. If the USSR were to be more successful than in the past in its efforts to reform economic management or to expedite the process of introducing new technology, its performance would exceed that which the production functions project. Finally, it should be noted that the various future trends in investment and military expenditures assumed in the report are not predictions but are projections to illustrate the effects of possible alternative programs.

The production functions cover both the non-agricultural non-service sectors of the economy as a whole and industry alone. Agriculture is excluded because year-to-year changes in production

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are affected so much by variation in weather as well as in the amount of land cultivated. Services such as education, health, and housing are excluded because output in these sectors is measured by the amount of inputs of either labor or capital; no separate measure of output exists.

The statistical basis for the production functions described in this report is found in CIA estimates of GNP originating in the non-agricultural and non-service sectors of the Soviet economy (or, alternatively, in industry) in 1950-68. The data on labor inputs (expressed in man-hours) and on capital services (reflecting annual average fixed capital stock) are derived almost entirely from published Soviet sources.

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The Pattern of Postwar Soviet Industrial Growth

1. Industrial growth was rapid from 1950 to 1955, slowed somewhat from 1955 to 1960, and slowed still more after 1960. The non-agricultural non-service sectors followed the same trends. Only industry is discussed here because it illustrates the general pattern. The slower growth from 1955 to 1960 is partly explained by the slower growth of labor, as measured in man-hours, because of the shortening of the workweek. Capital stock maintained a high rate of growth throughout the 1950s. From 1960 on, the slower growth of output was not accompanied by a correspondingly slower growth in inputs. These relationships can be represented by an index of factor productivity. Capital and labor are combined into a weighted index of inputs, and the ratio of the index of output to the index of combined inputs is the index of factor productivity. Table 1 below summarizes the rates of growth of industrial output, inputs, and factor productivity in the 1950s and the 1960s.

Table 1

Soviet Industry: Average Annual Rates
of Growth of Output, Inputs, and Factor Productivity a/

					Percent
<u>Period</u>	<u>Output</u>	<u>Capital Stock</u>	<u>Labor in Man-Hours</u>	<u>Combined Inputs <u>b/</u></u>	<u>Factor Productivity</u>
1951-60	9.8	11.4	2.2	4.9	4.7
1961-68	6.7	10.2	3.2	5.3	1.3

a. Based on data from Table 5.

b. Computed as a Cobb-Douglas production function, with labor and capital weights of 0.7 and 0.3, respectively.

2. The decline in the growth of factor productivity fully accounts for the slowdown in output because the growth of inputs is the same in both

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periods. Factor productivity to some degree is a measure of the contribution of new technology. Although many other factors such as the educational level of the labor force may be involved in factor productivity, there is considerable evidence that the new capital additions in the 1950s were more effective in upgrading industrial technology than the new capital in the 1960s.

3. However, the interpretation of the slowdown in output as a slowdown in factor productivity does not provide a systematic basis for projection. Should factor productivity be projected at the average rate of the last eight years or of a more recent period? There are grounds for arguing that the abrupt change in factor productivity after 1960 was due to the timing of the workweek reduction, which was completed in that year, and to the sudden acceleration of advanced weapons programs. If so, the underlying trend in factor productivity should be thought of as a more gradual decline.

4. Concurrent with the slowdown in the growth of production there has been a rapid growth of capital relative to labor. An increase in the capital/labor ratio associated with a slowdown in output describes a condition of rapidly diminishing returns to new capital stock and hence to investment. In industry, the result was that the average output-capital ratio fell steadily from 0.60 in 1951-55 to 0.56 in 1956-60, 0.46 in 1961-65, and 0.41 in 1966-68.

5. It is a reasonable hypothesis that diminishing returns will continue in the Soviet economy if capital continues to grow rapidly. In the ensuing sections, a production function is developed which incorporates this hypothesis.

Production Functions and Diminishing Returns to Capital

6. Over the past two decades, economists have studied economic growth in many countries in an intensive effort to untangle the separate contributions to growth of labor, capital, and new technology. These studies have proceeded by

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finding a production function -- that is, a mathematical relation between production and the factors that contribute to and presumably determine production. A widely used production function is one known as the Cobb-Douglas function. Its formula is:

$$Q = AK^bL^{(1-b)}$$

Where:

Q=production,
K=capital inputs, and
L=labor inputs

The coefficients *b* and $(1-b)$ are generally taken to be equal to the shares of total income going to capital and labor, respectively. The coefficient *A* incorporates primarily new technology, qualitative improvements in management and the labor force, good luck, and anything else not accounted for by quantitative changes in labor and capital. It is in effect a residual factor.

7. The Cobb-Douglas function fits the historical growth in the United States very well. For the period 1929-60, capital grew 2.0% a year; labor 0.7%, and production 3.1%. The shares of capital and labor in the total factor income in the United States have been computed at $b = 0.25$, and $(1-b) = 0.75$, respectively. Fitting the Cobb-Douglas function with these coefficients to actual values of output results in a rate of growth of *A* of about 2% per year.* The part of growth represented by *A* is attributed mostly to new technology but also partly to the increasing educational level of the labor force.

8. The Cobb-Douglas function works well for the United States because there has been no long-run tendency for the growth of output to slow down relative to capital and labor. In the case of the USSR there has been such a slowdown for

* *Richard R. Nelson, "Aggregate Production Functions and Medium-Range Growth Projections," The American Economic Review, Vol. LIV, no. 5, Sep 1964, p. 575.*

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industry (as shown in Table 1) and for GNP. The Cobb-Douglas function cannot duplicate this slowdown. If we put the function in terms of rates of growth over time for outputs and inputs, we obtain a very simple and useful form of the function:

$$\dot{Q} = \dot{A} + b\dot{K} + (1-b)\dot{L}$$

Where:

\dot{Q} , \dot{A} , \dot{K} , and \dot{L} are annual rates
of growth

Production growth, according to this formula, cannot slow down unless either \dot{A} or the growth in combined inputs falls off. In the USSR the rate of growth of production has declined while capital has continued to increase at a rate very much faster than the rates for either labor or production. In the framework of the Cobb-Douglas function the decline has been attributed almost entirely to a slowing of the rate of introduction of new technology.* Realistically, diminishing returns to capital are as plausible an explanation.

9. A different kind of production function was proposed a few years ago which allows for rapidly diminishing returns to capital and hence seems to fit the Soviet case. An increase in capital relative to labor in this formula results in a slowdown in growth of output even though both capital and labor continue to grow. This production function is named after some of its original proponents: the Arrow-Chenery-Minhas-Solow function, or the ACMS function for short.

10. In the context of Soviet economic growth, the crucial difference between the ACMS and the Cobb-Douglas functions is the role of capital implied by each. From each function we can derive an expression for the rate of change of production that results from a change in capital,

* A steady improvement in the quality of the labor force because of advancing levels of education accounts for a small part of \dot{A} . The rate of growth of average educational attainment has not declined significantly, so it has not contributed to the slowdown in the growth of \dot{A} .

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holding labor and the residual A unchanged. For the Cobb-Douglas function this expression is:

$$\frac{\text{Percentage change in } Q}{\text{Percentage change in } K} = b^*$$

The expression is constant. A given rate of growth of capital will always give the same rate of growth of output, assuming there is no change in the rates of growth of labor and the residual. If $b = 0.25$, as in the United States, then a 1% increase in capital stock, with no change in labor or the residual A , will yield a $\frac{1}{4}$ % increase in production.

11. The corresponding expression for the ACMS function, however, is:

$$\frac{\text{Percentage change in } Q}{\text{Percentage change in } K} = \frac{b}{b + (1-b)\left(\frac{K}{L}\right)^c}$$

Where: c is a coefficient equal to or greater than zero that determines how returns to capital diminish.**

If $c = 0$, the expression above reduces to b which is the same as for the Cobb-Douglas function. Mathematically, the Cobb-Douglas function is a special case of the ACMS function. If c is greater than zero and the capital-labor ratio K/L is increasing the response of production to a given growth in capital is reduced. How fast the returns to capital (in the sense of the rate of growth of production) diminish, depends on c . The larger c is, the smaller is the growth of production for a given growth in capital.

12. Studies of the United States have resulted in values of c of about one.*** This value would

* See p. 33 of the Appendix for the derivation of this formula.

** The general formula for the ACMS function is: $Q = A[bK^c + (1-b)L^{-c}]^{-\frac{1}{c}}$. See the Appendix for a description of the ACMS function and for the derivation of the formula relating the percentage change in output to a percentage change in capital stock.

*** Marc Nerlove, "Recent Empirical Studies of the CES and Related Production Functions," The Theory and Empirical Analysis of Production, National Bureau of Economic Research, 1967.

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give much more rapidly diminishing returns with rapidly growing capital than would the Cobb-Douglas function. However, in the United States capital has not grown very much faster than labor. What tendency there has been toward diminishing returns has been offset by advancing technology. It is not an accident that returns to capital have not declined steadily in the United States. In a market economy, investment decisions and hence the rate of growth of capital are governed by expected return. Historically, when production has slowed down, capital formation has also slowed down. In this case, the Cobb-Douglas function fits historical trends about as well as the ACMS function.

13. In the case of the USSR, capital has grown much faster than labor or production. Hence, any tendency toward diminishing returns should have an observable statistical effect. The value of $\frac{\text{Percentage change in } Q}{\text{Percentage change in } K}$, labor held constant, can be approximated by using changes in Q/L , production per man-hour of labor, and in K/L , capital per man-hour of labor.* These percentage changes by five-year periods for Soviet industrial production give the following ratios: for 1951-55, 0.91; 1956-60, 0.78; 1961-65, 0.45; 1966-68, 0.66. The ratio for 1961-65 seems very low, in part because of the effect of bad weather in agriculture which held down production in the light and food industries. Otherwise the downward trend is clear. Such a downward trend in returns to capital is not consistent with a Cobb-Douglas function but is consistent with an ACMS function.

Statistical Production Functions for the USSR

14. The hypothesis that the ACMS production function is more suitable than the Cobb-Douglas function for the analysis of Soviet growth experience was tested by fitting both functions to the Soviet data. The data used were for Soviet industry and for the non-agricultural non-service

* See the Appendix, Table 5, for data. The formula used was

$$\frac{\left(\frac{Q_1}{L_1} - \frac{Q_0}{L_0}\right) \left(\frac{K_1}{L_1} - \frac{K_0}{L_0}\right)}{\frac{1}{2}\left(\frac{Q_0}{L_0} + \frac{Q_1}{L_1}\right) \frac{1}{2}\left(\frac{K_0}{L_0} + \frac{K_1}{L_1}\right)}$$

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(NANS) sectors for the period 1950-68.* In each case the precise mathematical form of the production function was estimated statistically. The estimates of the Cobb-Douglas function, however, followed widely accepted practice in setting the coefficients for labor and capital equal to their shares in total income.**

15. The results of what seem to be the "best of show" in three categories of functions are shown in Figures 1 through 3, where the observed values of NANS and industrial output in 1950-68 are compared with the values of output predicted by the production functions that were fitted to the data on input and outputs.*** In Figure 1, the Cobb-Douglas function (I) represents a regression line fitted to the entire period. In Figure 2, the Cobb-Douglas function (II) represents two separate regression lines fitted to the two periods, 1950-60 and 1961-68. In Figure 3, an ACMS function is fitted to the whole period.

16. Cobb-Douglas I accounts for the post-war growth with an average annual growth of A of 3.3% for industry.† The deficiency of this approximation is clear from the chart. The growth predicted by the function is slower than actual growth in the first half of the period and faster in the second half. While actual growth of industry declined from nearly 10% in the first half to 6% in the second half, the growth predicted by the function maintains a steady rate of 8½% per year.

17. In Figure 2, Cobb-Douglas II matches the decline in growth because it is actually two functions separately fitted to the two periods. For the first period the rate of growth of A for industry is 4.7% per year and for the second period,

* The NANS sectors include industry, construction, transportation and communications, and trade and distribution.

** With an imputed interest charge of 8% on fixed and working capital, the shares turn out to be 0.70 and 0.30 for labor and capital, respectively -- for both the NANS sectors and industry. These shares resemble closely the shares frequently used in the analysis of Western aggregate production functions. For further discussion of the Cobb-Douglas weights, see the Appendix.

*** The Appendix presents a discussion of the estimating techniques and shows the full array of test statistics.

† In Figures 1 and 2, $A = ae^{\lambda t}$. The coefficient λ is the annual rate of growth of A .

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1%. In the functions the slowdown is accounted for entirely by the slowdown in A. A is recognizable as factor productivity for the two periods.*

18. The ACMS function in Figure 3 achieves a reasonable fit with a single function over the whole period. The value for c -- 2.28 for industry -- indicates very rapidly diminishing returns as the capital-labor ratio increases. In this function, A does not grow with time. The growth of output is sufficiently explained by K and L.

19. The relationships between inputs and outputs estimated for a Cobb-Douglas function and for an ACMS function differ greatly with respect to the predicted response of output to a change in capital stock. In 1950 and 1968 a 1% increase in capital stock results in the following percentage increases in predicted output:

<u>Function</u>	<u>NANS Sectors</u>		<u>Industry</u>	
	<u>1950</u>	<u>1968</u>	<u>1950</u>	<u>1968</u>
Cobb-Douglas I	0.30	0.30	0.30	0.30
ACMS	1.00	0.37	0.92	0.34

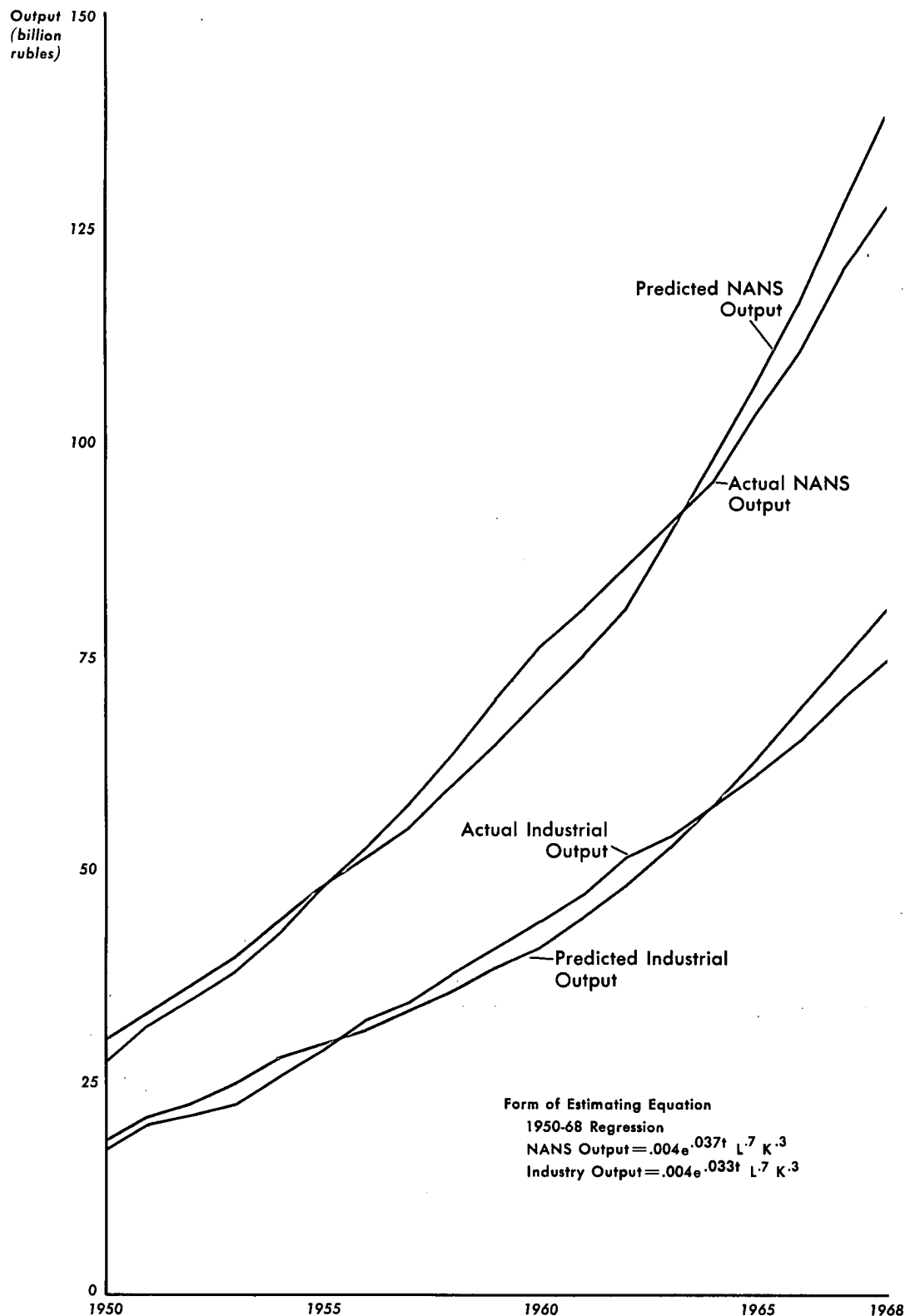
According to these estimates, the relative response in output to a change in capital stock is constant for the Cobb-Douglas function, and is decreasing over time in the case of the ACMS function. Under the ACMS formulation, the response of output to additions to capital stock, or the return on new investment, will continue to fall as long as capital stock grows more rapidly than labor. And the USSR is a classic example of a country where the capital-labor ratio has risen relentlessly since the war.

* The values for average annual growth in factor productivity shown in Figure 2 can and do differ somewhat from the values presented in Table 1 because they are derived by regression analysis instead of by dividing an index of output by a weighted index of inputs.

**USSR: Comparison of Observed Values of Output
in Non-Agricultural Non-Service (NANS) Sectors and in Industry
with Values Predicted by Cobb-Douglas Function I**

Figure 1

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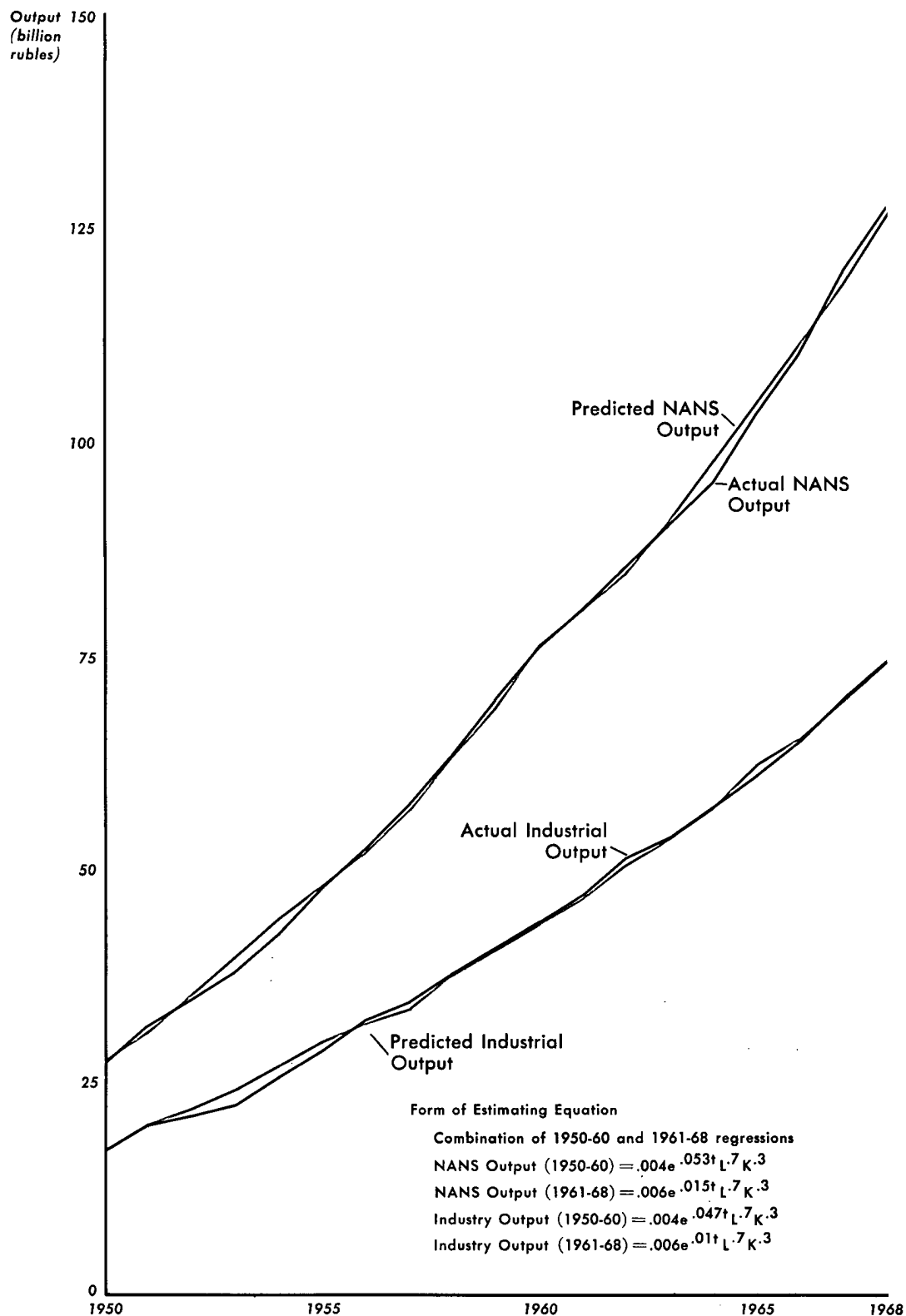
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Figure 2

**USSR: Comparison of Observed Values of Output
in Non-Agricultural Non-Service (NANS) Sectors and in Industry
with Values Predicted by Cobb-Douglas Function II**

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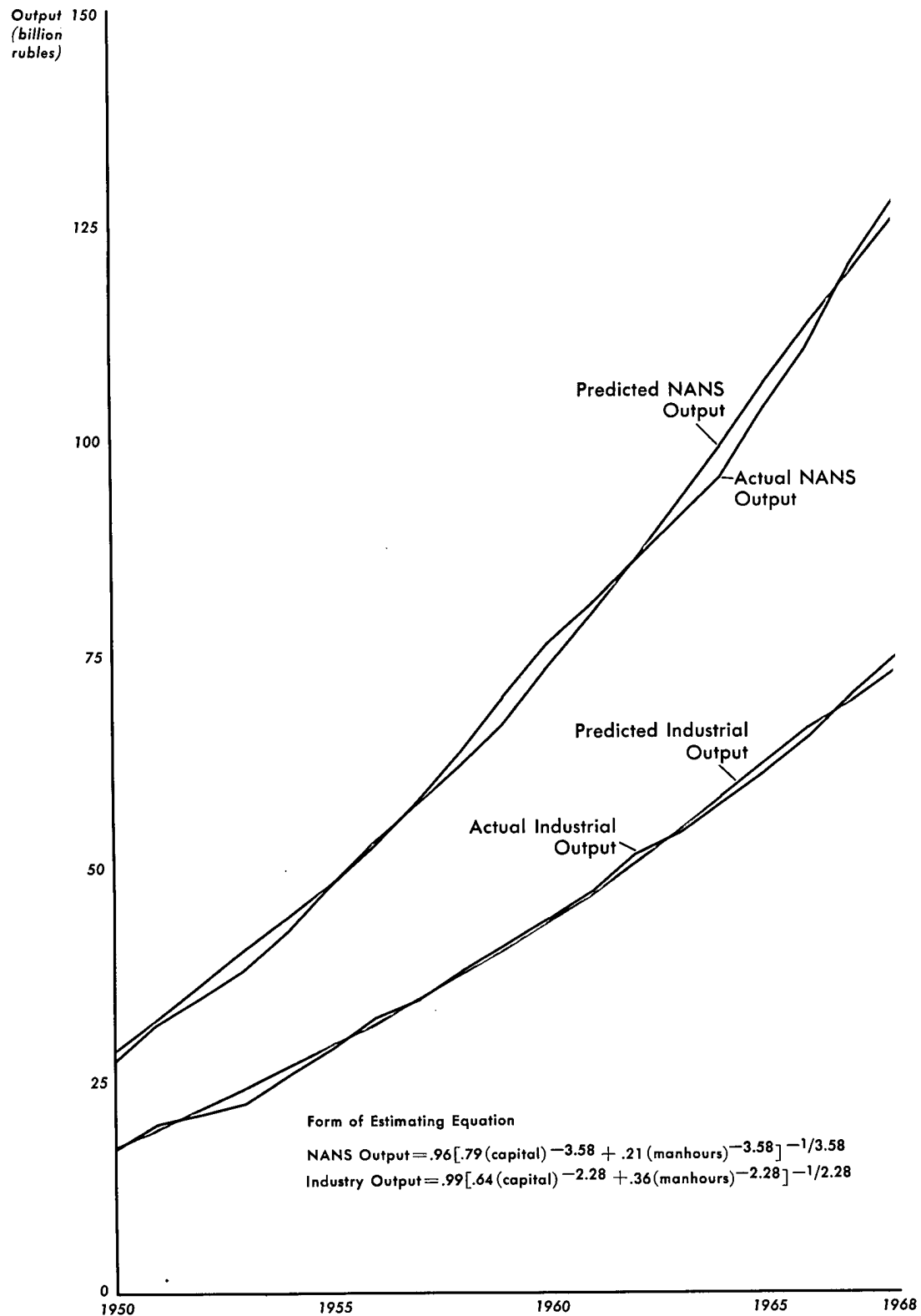
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Figure 3

**USSR: Comparison of Observed Values of Output
in Non-Agricultural Non-Service (NANS) Sectors and in Industry
with Values Predicted by an ACMS Function**

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20. Estimating equations based on both versions of the production function provide a good fit to the observed data on output. The share of the changes in observed output (the R^2) that is accounted for by the effect of changes in labor and capital inputs on the estimating equations is:

	Cobb-Douglas I (Figure 1)	Cobb-Douglas II (Figure 2)	ACMS (Figure 3)
NANS	0.994	0.999	0.997
Industry	0.983	0.999	0.999

The fact that the Cobb-Douglas I function performs somewhat more poorly than the ACMS function over the whole period seems to be due to an abrupt change in the growth of factor productivity in the 1960s compared with the 1950s.

21. On the basis of the fits shown in Figures 1, 2, and 3, there is no overwhelming reason to choose one function over another when trying to project output into the future on the basis of trends in investment. There are, however, at least two reasons for preferring the ACMS function to Cobb-Douglas II as a basis for projections. First, the ACMS function is the most general function, and, in the course of the statistical estimation, the equation relating output to inputs could have turned out to be of the Cobb-Douglas type -- but it did not. Second, the Cobb-Douglas I equation, which was based on the 1950-68 period as a whole, produced a curve of predicted output (as in Figure 1) that understates growth in the early and overstates growth in the latter part of the 1950-68 period -- that is, it fails to generate a slowdown in growth.

22. It is impossible to choose statistically between the Cobb-Douglas II functions and the ACMS functions for industry or NANS. The choice must be made on a priori grounds derived from the relative roles of new technology and of diminishing returns in the Soviet economy. This is discussed below.

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New Technology and Diminishing Returns

23. In the ACMS function for industry and NANS the A term shows almost no change over time because λ is not significantly different from zero.* In the ACMS functions for the USSR, technical progress, to the extent it occurred, is tied to changes in inputs of labor or capital stock instead of being a factor contributing to output over time regardless of changes in inputs. In the Cobb-Douglas functions, on the other hand, A is a residual (because the weights for labor and capital are specified) which does grow with time. If a choice has to be made between the two functions as a guide to what might happen in the future, there are certain features of Soviet economic development that favor the ACMS function.

24. The notion of diminishing returns is closely related to the substitution of capital for labor.** When capital grows more rapidly than labor, returns to capital do not have to decline, however, if there is an offsetting change in the production method -- that is, technology. In practice, most new technology is embodied in new capital stock, and most investment contains some degree of relatively new technology. This steady stream of new technology, in an efficient environment, reorganizes production processes and thereby increases output while saving labor, raw materials, and land. The existence of actual diminishing returns means that the new plant and equipment is very much like the old plant and equipment already operating, or that the potential of the new technology embodied in the new capital is not being realized. If the appropriate reorganization of the production process is not carried out (in full), then production is not increased (fully) and labor is not released (as much as could be). This is the situation that prevails in much of the Soviet economy. The combination of resources in any industry, enterprise, or process in the USSR is decided by bureaucrats who may or may not know or care what the least cost combination is.

* See the fourth footnote on p. 11.

** For a discussion of the elasticity of substitution in the ACMS function, see the Appendix.

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25. The Soviet economy is notorious for its wasteful use of labor. There are many reports of particular plants, including those purchased from the West, where employment is very much larger than Western labor requirements would be. Average labor productivity by industry or sector tells the same story. The barriers to the substitution of capital for labor are formidable. Managers find it prudent to hoard workers, and the government and labor unions strongly oppose unemployment. In addition, there is no automatic system for transferring resources which are saved from one use to another. Labor-intensive industries do not expand on demand from consumers but on command from planners.

26. The resistance of enterprise managers to the introduction of new products is equally notorious. Any new development whose cost is in one sector but whose benefits are in another has hard sledding. This is illustrated in the slow development of thin rolled-steel sheet, of metal forming processes in place of casting and metal cutting, and of plastics and synthetic fibers. The failure of computer manufacturers to provide maintenance, training, and programming services for their customers is another case in point. The effect of these rigidities is a pervasive tendency of the system to reproduce itself in the same mix of output and the same pattern of investment. Khrushchev described the phenomenon accurately when he said the proponents of the expansion of crude steel capacity were like "blind horses following a rut."

27. In the 1950s the average quality of capital stock increased rapidly as the USSR made up its war losses. The technology at hand was many years younger than that incorporated in prewar plant and equipment. As capital stock accumulated, the process of lowering its average age slowed and finally stopped. The inordinate delays in planning, design, and construction of new production facilities brought this to pass sooner rather than later. A factory which has required ten years from initial funding to initial production -- as many Soviet factories do -- cannot be called technologically new.

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28. The suitability of the ACMS function for describing Soviet experience rests not only on its statistical fit but on the nature of the Soviet system for using resources. The difficulty of substituting capital for labor by innovation and by transferring resources from one use to another which is characteristic of the Soviet bureaucratic economy is accurately described by the ACMS functions derived here.

Projections of Soviet Growth Under Alternative Investment Programs

29. Soviet economic managers have been unable or unwilling in the 1960s to maintain previous rates of growth of capital stock. Meanwhile, echoes of a policy debate over future investment plans are reaching the West as the guidelines of the 1971-75 plan are being drafted. On the assumption that the ACMS functions estimated above provide a basis for projection, alternative investment programs in the non-agricultural non-service and industrial sectors provide some interesting comparisons.

30. The projection period is 1969-80. In this period, a reasonable range for the average growth in labor inputs in industry and in the NANS sectors is 2% to 3% per year. A fairly wide range of investment programs would result in rates of growth of capital stock of from 5% to 10% per year. In industry, for example, with the present size of capital stock and current retirement rates, the USSR could achieve a 5% annual increase in capital stock with an average annual rate of growth of investment of only 1½%. An average annual growth in capital stock of 10% per year would require an increase in investment of 10½% per year. The lower rate is less than any experienced thus far in the USSR while the higher figure is well above the average annual rate of growth of capital stock in industry in the 1960s.

31. Surprisingly, the alternative rates of growth of capital stock make very little difference in the terminal value of output in 1980 or in the total value of output in 1969-80 (see Table 2). On the other hand, the response of

Table 2

USSR: Values of Output Projected by an ACMS Function
Under Alternative Assumptions About Inputs
1969-80

Assumed Average Annual Growth in Inputs	Projected Output					
	NANS Sectors			Industry		
	Billion Rubles		Average Annual Rate of Growth 1969-80 (Percent)	Billion Rubles		Average Annual Rate of Growth 1969-80 (Percent)
	1980	1969-80 <i>a</i> /		1980	1969-80 <i>a</i> /	
Man-hours, 2%; capital stock, 5%	173	1,814	2.7	103	1,062	2.8
Man-hours, 2%; capital stock, 10%	180	1,887	3.1	110	1,121	3.4
Man-hours, 3%; capital stock, 10%	202	2,007	4.0	122	1,189	4.3

a. Cumulative.

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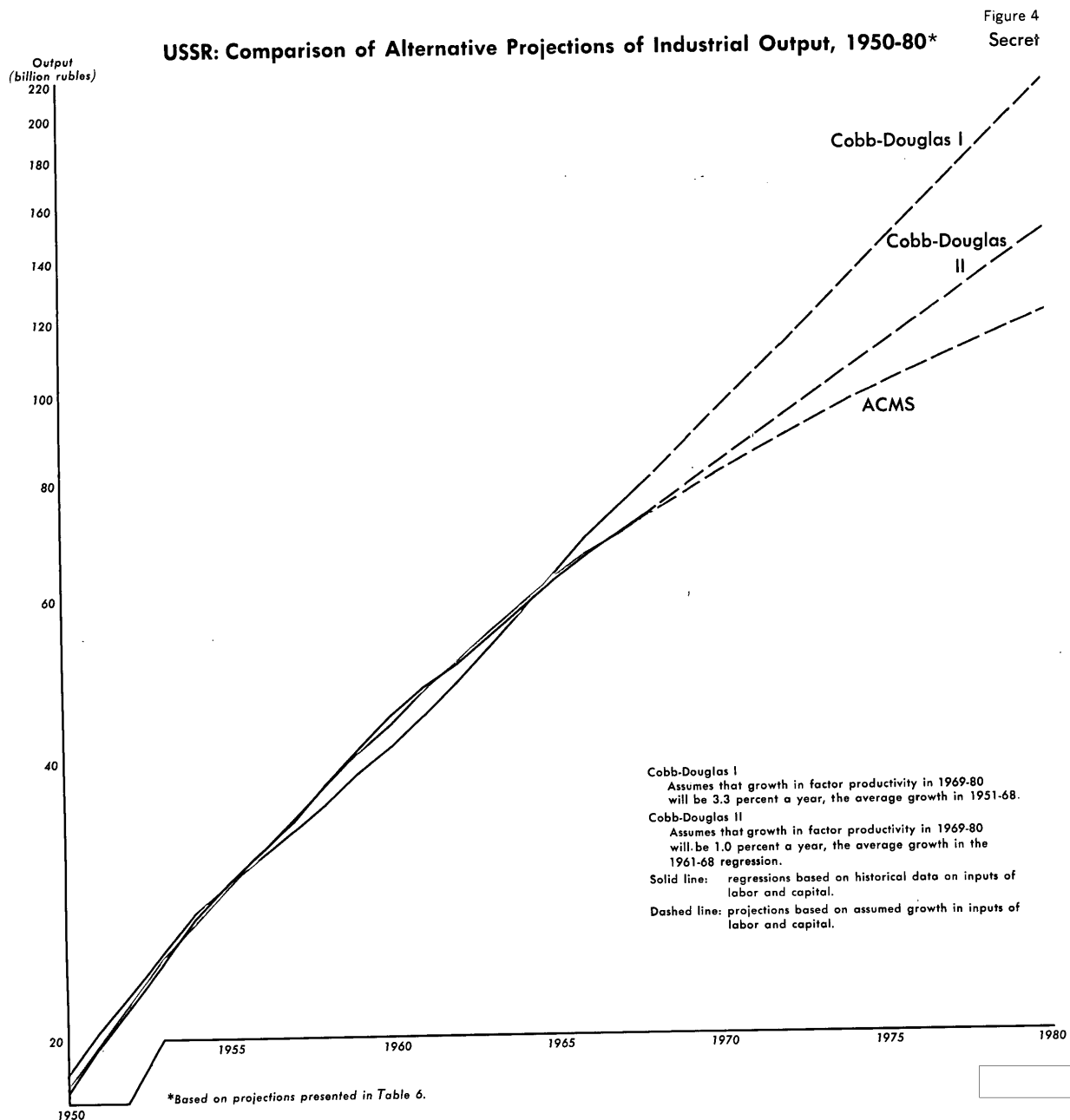
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output to a change in the rate of growth of labor inputs is much stronger. For example, in the NANS sectors, when the annual growth in man-hours is fixed at 2% per year, an increase in the rate of growth of capital stock from 5% to 10% per year raises the value of output projected for 1980 by just 7 billion rubles. In contrast, when the annual growth in capital stock is held at 10% per year, an increase in the average annual growth of man-hours from 2% to 3% per year results in a gain of 22 billion rubles in the output projected for 1980.

32. The results of this simulation are actually predetermined by the fact that the ACMS function estimated in this report shows such rapidly diminishing returns to capital. Returns to new investment diminish very rapidly when capital stock grows much faster than inputs of labor. If the same projection were made with the Cobb-Douglas function, with its built-in constancy of returns to investment, the effect on output of the growth in capital stock assumed above would be substantially greater. For example, assuming a growth in capital stock and labor inputs at the upper limit of the range outlined above,* the NANS sector output in 1980 would be projected at 202 billion rubles with the ACMS function, but at 391 billion rubles with the Cobb-Douglas I function (assuming that factor productivity increased at the same rate in 1969-80 as it did on the average in 1951-68). Using the same upper limit on the growth of inputs as above, industrial output in 1980 projected with the help of an ACMS function is 122 billion rubles and with a Cobb-Douglas I function, 216 billion rubles.

33. The very different rates of growth of output in the NANS sectors and in industry that are implied by the three functions can be seen in Table 3. Figure 4 shows the projections of industrial output given by the Cobb-Douglas and the ACMS production functions when labor and capital inputs grow at 3% and 10% per year, respectively, in 1969-80. These projected rates of growth of inputs are both faster than the actual rates in industry or NANS in the last five years. One of the Cobb-Douglas projections (I) assumes that factor productivity grows at 3.3% a year, as it

* At 10% and 3% a year, respectively.



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Table 3

USSR: Alternative Projections of Output a/
1969-80

	GNP in NANS Sectors in 1980 (Billion Rubles)	Average Annual Percentage Rate of Growth of NANS Output	
		1969-75	1976-80
Cobb-Douglas I <u>b/</u>	391	9.0	9.0
Cobb-Douglas II <u>c/</u>	276	6.7	6.7
ACMS	202	4.5	3.4

	Industrial Output in 1980 (Billion Rubles)	Average Annual Percentage Rate of Growth of Industrial Output	
		1969-75	1976-80
Cobb-Douglas I <u>d/</u>	216	8.6	8.6
Cobb-Douglas II <u>e/</u>	150	6.1	6.1
ACMS	122	4.7	3.8

a. Assuming that in 1969-80 inputs of labor will grow by 3% a year and inputs of capital stock by 10% a year. See Table 6 for supporting data.

b. Assuming that the average annual growth of factor productivity (A) will be 3.7% a year, as in the 1951-68 regression.

c. Assuming that the average annual growth of factor productivity (A) will be 1.5% a year, as in the 1961-68 regression.

d. Assuming that the average annual growth of factor productivity (A) will be 3.3% a year, as in the 1951-68 regression.

e. Assuming that the average annual growth of factor productivity (A) will be 1.0% a year, as in the 1961-68 regression.

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did in the regression covering the period 1951-68; the other Cobb-Douglas projection (II) assumes that factor productivity increases by only 1.0% a year, as in the regression covering the period 1961-68. The outputs predicted by the two variants of Cobb-Douglas functions each lie on a straight line -- signifying a constant rate of growth. The ACMS function, however, projects a continuously declining rate of growth -- represented by a line of constantly decreasing slope.

34. In Table 3 it is quite clear that the projections of output relying on Cobb-Douglas function II, which assumes a continuation of the factor productivity of the past eight years, yields a much smaller stream of output than the projection relying on Cobb-Douglas function I. The growth of output generated by the ACMS function is still less.

35. Although a great deal of caution is required in extrapolating the results of any regression exercise, the ACMS projection and the projection implied by the Cobb-Douglas II function carry a message that Soviet planners seem already to be heeding. The projections discussed above embody the rates of technical progress achieved recently. If the ACMS projections genuinely reflect the long-term prospects attending accelerated investment programs, then current Soviet emphasis on spurring technological progress and the laments about a technological gap between the USSR and the West are certainly justified. The ACMS function for industry, for example, projects an average rate of growth of 4.3% a year in 1969-80 compared with an estimated 6.7% a year in 1961-68 and 9.6% a year in 1951-60. And this rate of growth assumes an average annual increase in inputs of labor and capital that the USSR will be hard put to achieve.

Implications for Policy

36. The implication of the preceding analysis is that more investment will not solve the USSR's growth problem. Diminishing returns to investment of the degree suggested above mean that efforts to push up the rate of growth of investment will have only a meager payoff. In the preceding section,

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projections made with the help of the ACMS function indicate that doubling the rate of growth of investment from 5% to 10% a year would raise the annual rate of growth of output by less than 0.5%.

37. Even in the event that the substitutability of capital for labor is really greater than that implied by the ACMS function, the weight of the evidence is that diminishing returns to investment in the USSR are too strong to be accounted for by a Cobb-Douglas function. In the Soviet case, the sharply rising capital-labor ratio makes the effect of diminishing returns quantitatively significant.

38. A direct approach to the problem would be to try to increase the growth of the labor force. But the USSR is severely limited in this direction by demographic circumstances and by its past success in keeping the labor force participation rate at such a high level. The best hope it has of increasing the supply of labor to the nonagricultural sectors is by improving productivity in agriculture, where one-third of the labor force is still tied down. A very remarkable transformation of agricultural management would have to occur before labor could be released rapidly without adversely affecting the output of agriculture.

39. Another much discussed possibility is the transfer of resources from defense. In the light of the analysis above, a simple shift of resources to investment is unpromising. The proportion of Soviet GNP at factor cost that has been allotted to military-space programs has been fairly stable, between 7% and 8% from 1965 to 1969, while the absolute value of these outlays has increased about one billion rubles per year. Amounts of this magnitude are very small in comparison with investment, and diminishing returns would make the production effects smaller still. If defense were held constant at the 1969 level and an annual growth of one billion rubles were reallocated to investment in industry, the increment in investment would be one billion in 1970 and six billion in 1975. The rate of growth of capital stock in industry would be raised by about 1%. According to the ACMS function, production in industry would

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accelerate by about 0.1% and according to the Cobb-Douglas II function about 0.2%.

40. It has been argued that the transfer of resources from defense to civilian uses would increase productivity directly. Defense has had prior claim on the highest quality resources, machines, materials, and men. In particular, some of the high-quality engineers and skilled labor, plus high-grade materials, would certainly help in civilian investment programs. However, the quantities of these resources are small compared to civilian investment, and effective transfer would require some very complex reorganization and adjustment. It is hard to see, for example, how a transfer of half a billion rubles worth of goods and services per year could significantly upgrade the efficiency of an annual capital investment in the NANS sectors of 30 billion rubles. Moreover, given the existing bureaucratic organization of investment, it is doubtful if these high-grade resources would get to the right uses in most of the cases.

41. The effect of transferring resources from military research and development (R&D) to civilian R&D is potentially more profitable because the volume of resources that might be conceivably shifted is large compared to current expenditures on civilian R&D, provided that Soviet planners were willing to go as far as to halt the growth of military R&D. In this case, the critical question is whether the environment for civilian R&D would be as conducive to innovation and application of the innovations as is the military environment. In the military environment, R&D, as well as investment, now has the advantage of (a) a discriminating and demanding consumer with the power to enforce its demands; (b) a priority which guarantees that supply schedules are met; and (c) less bureaucratic red tape and Party interference. The reforms that would be necessary to transfer these advantages to civilian R&D are hard to visualize.

42. The discussion above suggests that the USSR will have to choose between accepting a lower (and possibly still declining) rate of

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growth and attempting to improve the efficiency of the system on a broad front. For some time, the USSR has been tinkering with the administration of civilian R&D and with the defects of project planning. Experimentation with different methods of organizing construction and the design and installation of new equipment has a still longer history. Based on the results thus far, it is arguable that a more far-reaching reorganization of the administration of the economy is required. The difficulty is that neither the Soviet leaders (nor anyone else) can suggest a sure-fire program of reform that on the one hand will spur economic progress and on the other insure the degree of central control that the leadership considers to be essential.

Conclusions

43. The finding of this report is that Soviet economic growth since 1950 is best described by a production function in which strongly diminishing returns to new investment occur. This function, known as the ACMS function, fits the growth of the Soviet industrial and non-agricultural non-service sectors better than a Cobb-Douglas production function of the kind formerly used. In trying to achieve the highest possible volume of investment, Soviet economic policy has forced the capital-labor ratio continuously upward, and this strategy accentuates the effect of diminishing returns. Under these conditions, the ACMS production function estimated for the USSR -- with its relatively low substitutability of capital for labor -- generates a gain in output per unit increase in capital stock that falls off sharply over time. This pattern of growth accurately matches the observed Soviet slowdown since the 1950s.

44. If the relation of output to inputs in the USSR is of the character described by the ACMS function, the situation confronting the Soviet leadership is indeed discouraging. A continuation of the growth of man-hours and capital

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stock at the same rate as in the 1960s would result in a projected average annual rate of growth of output in the non-agricultural non-service sector of only 4.0% a year during 1969-80 -- far less than the 7.0% a year achieved in 1961-68 or the 8.6% in 1951-68. In a turnabout from its earlier economic history, the USSR would have to deal with a series of planning periods in which the growth of the labor force -- not the growth of capital stock -- is the real constraint on the rate of growth of output.

45. Should returns to investment -- or what amounts to the same thing, the substitutability of capital for labor -- actually be somewhat higher than the value projected by the ACMS function, the prospects would be brighter. Nevertheless, diminishing returns to new investment would be a serious problem for the leadership over a wide range of plausible functions. Studies of Western economies have found the substitutability of capital for labor to be lower than that inherent in the Cobb-Douglas production function, so a like finding for the USSR is credible.

46. Given a diminishing rate of growth of output with respect to capital, a transfer of a billion rubles from other end uses to investment was found to have a smaller and smaller effect on growth over time. This would be true for a simple transfer of funds from defense to investment. But high-quality resources, particularly scientific and technical manpower, now employed in defense might have a more than proportional effect on growth. Even so, it is doubtful if the potential of these resources could be fully realized without some drastic shake-up in the management of civilian R&D and investment.

47. The implications of such strongly diminishing returns to new investment for Soviet policy are pointed. Having assembled a huge stock of capital, the USSR needs to adopt a different strategy for growth. According to Simon Kuznets,

Modern economic growth is distinguished by the fact that the rate of rise in per capita product

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was due primarily to improvements in quality, not quantity of inputs -- essentially to greater efficiency -- traceable to increases in useful knowledge and better institutional arrangement for its utilization.*

48. A change of priorities favoring a higher rate of capital formation will not insure even a continuation of present rates of economic growth. While the USSR recognizes that it is behind the West technologically and that it is not closing the gap, the policies necessary to spur technological progress are not obvious. The discussion above suggests that the USSR will have to choose between accepting a lower (and possibly still declining) rate of growth and attempting to improve the managerial efficiency of the system on a broad front. The dilemma for Soviet leaders is that no one has suggested a sure-fire program of reform that will spur economic progress and also insure the degree of central control that the leadership considers to be essential.

* Modern Economic Growth -- Rate, Structure, Spread, 1966, p. 491.

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STATISTICAL APPENDIXDescription of the Production Functions

Two forms of aggregate production functions were estimated for this report, as follows:

(1) Cobb-Douglas model:

$$Q = ae^{(\lambda t)} K^{(b)} L^{(1-b)} e$$

(2) Constant elasticity of substitution model (the ACMS function):

$$\text{Where: } Q = ae^{(\lambda t)} [bK^{-c} + (1-b)L^{-c}]^{\frac{1}{c}} e^*$$

Q denotes output,

K denotes capital,

L denotes labor, and

e is the error term.

The Cobb-Douglas function is a special case of the more general ACMS function. The ACMS, or constant elasticity of substitution, production function allows the elasticity of substitution to vary between zero and infinity, although it maintains a constant value for a given function. The Cobb-Douglas function restricts the value of the elasticity of substitution to one. [In the ACMS function described above, the elasticity of substitution (σ) is related to c by: $c = (\frac{1}{\sigma}) - 1$]. The elasticity of substitution measures the ease with which inputs can be substituted for each other in production. When $\sigma = 0$, production takes place in fixed proportions. This means that if, for

* The above formulation indicates Harrod neutral technological change. When the value of λ is determined by an unconstrained estimate, it assumes a negative value which is exceedingly difficult to analyze. In addition, it is impossible to choose which way the technological change parameter enters the function, except arbitrarily. Therefore, it was decided to constrain λ , in the ACMS case, to zero.

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example, capital is increased but labor is not, there would be no increase in output. When σ (the elasticity of substitution) is infinite, the two inputs are perfect substitutes in the production process. This implies that they can be substituted for one another, in a constant proportion, without affecting output.

The Cobb-Douglas parameters a and λ were estimated by ordinary least squares following a logarithmic transformation of the variables. The parameters b and $(1-b)$ are the elasticities of output with respect to capital and labor, respectively.* Assuming that all inputs are paid the value of their marginal products in the base period these elasticities are equal to each input's proportionate share of value added (in the base period). The resulting R^2 is computed for the transformed variables. The ACMS parameters were estimated by a nonlinear regression package that combines the method of "steepest descent" and the "gradient method." The resulting R^2 is estimated from the residuals.

The data used are described in another section of this Appendix. The values obtained for the parameters and the test statistics are presented below.

Table 4
Cobb-Douglas Function

Model	<i>a</i>	λ	<i>R</i> ²	<i>t</i> Ratio:		NOBS
				<i>a</i>	<i>b</i>	
Cobb-Douglas I						
NANS	-5.5009 <i>a</i> / (0.02666)	0.0371 (0.00234)	0.937	-206 <i>a</i> / (0.02666)	15.85	19
Industry	-5.4235 <i>a</i> / (0.02566)	0.0328 (0.00225)	0.926	-211 <i>a</i> / (0.02566)	14.59	19
Cobb-Douglas IIA 1950-60						
NANS	-5.5887 <i>a</i> / (0.01107)	0.05304 (0.00163)	0.99	-504.96 <i>a</i> / (0.01107)	32.51	11
Industry	-5.5014 <i>a</i> / (0.01318)	0.04669 (0.00194)	0.985	-417.29 <i>a</i> / (0.01318)	24.02	11

* See Cobb-Douglas weights in this Appendix.

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Table 4
(Continued)

Model	<i>a</i>	λ	<i>R</i> ²	<i>t Ratio:</i>		NOBS
				<i>a</i>	<i>b</i>	
Cobb-Douglas IIB 1961-68						
NANS	-5.1735 <i>a</i> / (0.02338)	0.01523 (0.00149)	0.946	-221.29 <i>a</i> / (0.00149)	10.21	8
Industry	-5.0704 <i>a</i> / (0.0279)	0.00958 (0.00178)	0.83	-181.8 <i>a</i> / (0.00178)	5.38	8

a. Natural log of *a*.ACMS Function:

The results of the nonlinear regression for the ACMS do not produce the same test statistics as do the ordinary least squares results. The individual confidence limits for each parameter (on a linear hypothesis) reflecting a range of plus or minus two standard deviations (covering 95.4% of the possible range) are available for each parameter estimate.

Model	<i>a</i>	<i>b</i>	<i>c</i>	<i>R'</i>	NOBS
ACMS (Constant elasticity of substitution)					
NANS	0.9639 (0.946-0.982)	0.7862 (0.724-0.848)	3.5788 (2.23-6.87)	0.9967	19
Industry	0.9875 (0.975-1.00)	0.6389 (0.615-0.662)	2.2808 (1.74-3.10)	0.9986	19

The numbers in parentheses are standard errors for the Cobb-Douglas models and the confidence limits for the CES model. The *R'* is the proportion of the variation in the dependent variable explained by the linear influence of the independent variables. The standard error of the estimator is the standard deviation of the sampling distribution of the estimator. The standard error thus gives some indication of the precision of the estimator. The *t* ratio gives a test statistic incorporating the standard error of the estimator and is used to test the hypothesis that the given parameter is equal

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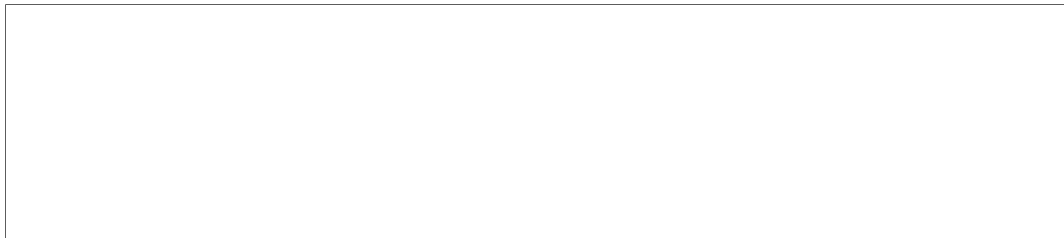
to zero. For 19 observations, if the absolute value of t is greater than 2.861, the hypothesis can be rejected at the 0.005 level of significance. The data show all parameters have t ratios greater in absolute value than 2.861.

Description of the Data

The data used in estimating the production functions described above are presented in Table 5. The values for annual output in industry and in the non-agricultural non-service sectors represent estimates of the ruble value of value-added in these sectors at 1960 factor cost prices.* Trends in capital services are assumed to be identical with the trends in the average of beginning-of-year and end-of-year values of productive fixed capital, as reported in official Soviet statistical handbooks. The estimates of man-hours worked in industry and in the non-agricultural non-service sectors are based, first, on reported data on average annual employment, and, second, on estimates of changes in hours worked per year as a result of reductions in the length of the workweek and the increasing number of holidays and leave days given to Soviet workers.

Alternative Projections of Output

With the aid of three of these production functions, alternative series for predicted output can be generated. The alternative projections are set out in Table 6, and the nature of the assumptions involved is explained in the footnotes to the table.



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Table 5

USSR: Data Used in Estimation of Production Functions

Year	Industry			Non-Agricultural Non-Service Sectors		
	Average Annual Capital Stock (Billion Rubles)	Man-Hours (Million)	Output (Billion Rubles)	Average Annual Capital Stock (Billion Rubles)	Man-Hours (Million)	Output (Billion Rubles)
1950	28.5	33,730	17.49	52.6	56,954	28.01
1951	32.0	35,650	20.15	59.2	59,651	31.98
1952	36.0	36,955	21.45	66.8	61,733	34.54
1953	40.0	38,514	23.37	74.2	64,068	37.85
1954	44.5	40,368	26.11	81.6	66,866	42.60
1955	49.5	41,098	29.45	89.8	68,540	47.84
1956	54.5	40,984	32.11	98.9	68,771	52.55
1957	60.6	40,927	34.50	108.6	69,506	57.75
1958	67.5	41,655	37.93	117.8	71,730	63.71
1959	75.5	42,032	41.34	128.3	73,395	69.98
1960	84.0	41,944	44.45	142.5	73,858	75.68
1961	94.5	42,534	47.66	158.0	74,488	80.19
1962	105.5	43,391	51.59	175.0	75,973	85.89
1963	117.0	44,935	54.37	194.2	78,440	90.50
1964	130.0	46,858	57.65	214.2	81,566	96.42
1965	143.5	48,868	61.31	235.6	84,995	104.13
1966	157.0	50,688	65.65	257.6	87,870	111.50
1967	170.0	52,297	70.53	279.1	90,767	120.34
1968	183.0	54,117	74.51	301.9	94,050	127.86

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Table 6

USSR: Comparison of Alternative Projections of GNP a/

Billion Rubles

Year	Non-Agricultural Non-Service Sectors			Industry		
	Cobb-Douglas Function		ACMS Function	Cobb-Douglas Function		ACMS Function
	A b/	B c/		A b/	B c/	
1950	29.7	27.6	28.6	18.4	17.3	17.5
1951	32.9	31.2	32.2	20.5	19.5	19.6
1952	36.3	34.9	36.2	22.5	21.7	21.9
1953	39.9	39.0	40.1	24.7	24.1	24.2
1954	43.9	43.6	44.0	27.2	27.0	26.7
1955	47.7	48.1	48.3	29.4	29.5	29.3
1956	51.1	52.3	52.8	31.2	31.8	31.7
1957	54.9	57.2	57.5	33.3	34.4	34.5
1958	59.7	63.2	62.0	35.9	37.7	37.6
1959	64.6	69.4	66.9	38.6	41.1	40.9
1960	69.5	75.9	73.0	41.2	44.4	43.9
1961	74.9	79.9	79.0	44.5	47.9	47.4
1962	81.2	84.8	85.4	48.2	50.7	50.9
1963	88.9	90.9	92.2	52.7	54.1	54.6
1964	97.7	97.7	99.2	57.8	58.1	58.6
1965	107.3	105.0	106.4	63.4	62.2	62.7
1966	117.1	112.1	112.9	69.1	66.2	66.5
1967	127.4	119.3	119.1	74.7	70.0	69.9
1968	138.7	127.1	125.5	80.8	74.0	73.3
1969	151.2	135.6	132.2	87.8	78.5	77.2
1970	164.9	144.6	138.9	95.3	83.2	81.1
1971	179.8	154.2	145.3	103.4	88.3	85.1
1972	196.0	164.5	151.8	112.3	93.6	89.0
1973	213.6	175.5	158.1	121.9	99.3	93.0
1974	232.9	187.2	164.4	132.3	105.3	97.0
1975	253.9	199.7	170.6	143.6	111.7	101.0
1976	276.8	213.0	176.7	155.9	118.5	105.0
1977	301.8	227.2	182.9	169.3	125.6	109.1
1978	329.0	242.3	189.2	183.7	133.3	113.2
1979	358.7	258.4	195.5	199.5	141.3	117.4
1980	391.0	275.7	201.8	216.5	149.9	121.6

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Table 6

USSR: Comparison of Alternative Projections of GNP a/
(Footnotes)

- a. Based on observed inputs of man-hours and capital stock in 1950-68 and assumed average annual increases in man-hours and capital stock of 3% and 10%, respectively, in 1969-80.*
- b. Based on the Cobb-Douglas function derived from observations of output and inputs in 1950-68.*
- c. Based on a combination of (1) the values for 1950-60 predicted by a Cobb-Douglas function derived from data for 1950-60; (2) the values for 1961-68 predicted by a Cobb-Douglas function derived from data for 1961-68; and (3) the values for 1969-80 predicted by the Cobb-Douglas function derived from data for 1961-68 and the projected inputs cited above.*

Cobb-Douglas Weights

(1) The general Cobb-Douglas function
 $Q = AK^b L^{(1-b)}$ is used for the following presentation.

(2) The marginal products of capital and labor are:

$$\frac{\partial Q}{\partial K} = bAK^{(b-1)}L^{(1-b)}$$

$$\frac{\partial Q}{\partial L} = (1-b)AK^b L^{-b}$$

(3) If capital and labor are paid the value of their marginal products their total shares will be:

For capital $(bAK^{(b-1)}L^{(1-b)})K$

For labor $((1-b)AK^b L^{-b})L$

(4) However, $(bAK^{(b-1)}L^{(1-b)})K = \left(\frac{bQ}{K}\right)K = bQ$,

and $((1-b)AK^b L^{-b})L = \left(\frac{(1-b)Q}{L}\right)L = (1-b)Q$.

Thus the shares of capital and labor exhaust total product, since $bQ + (1-b)Q = Q$.

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(5) It remains to be shown that b and $(1-b)$ equal the elasticities of output with respect to capital and labor respectively.* The output elasticity of capital is,

$$\frac{\partial Q}{\partial K} \cdot \frac{K}{Q} = \left(bAK^{(b-1)}L^{(1-b)} \right) \frac{K}{Q} = \frac{bQ}{Q} = b$$

Similarly for labor,

$$\frac{\partial Q}{\partial L} \cdot \frac{L}{Q} = \left((1-b)AK^bL^{-b} \right) \frac{L}{Q} = \frac{(1-b)Q}{Q} = (1-b)$$

(6) Thus the percentage share of output going to capital is b , which is also the elasticity of output with respect to capital. Similarly, $(1-b)$ is the percentage share of output accruing to labor, and $(1-b)$ is also the elasticity of output with respect to labor.

Derivation of the Elasticity of Output
with Respect to Capital, CES Function

The elasticity of output of the CES function with respect to capital can be derived as follows:

$$Q = A \left(bK^{-c} + (1-b)L^{-c} \right)^{-\frac{1}{c}}$$

$$\frac{\partial Q}{\partial K} = -\frac{1}{c} A \left(bK^{-c} + (1-b)L^{-c} \right)^{-\left(\frac{1}{c} + 1\right)} (-cbK^{-c-1})$$

$$\frac{\partial Q}{\partial K} = \frac{QbK^{-(c+1)}}{(bK^{-c} + (1-b)L^{-c})}$$

$$\frac{\partial Q}{\partial K} \cdot \frac{K}{Q} = \frac{bK^{-c}}{(bK^{-c} + (1-b)L^{-c})} = \frac{b}{(b + (1-b)\left(\frac{K}{L}\right)^c)}$$

* The elasticity of output with respect to a given factor (labor or capital) is equal to the percentage change in output divided by the percentage change in the factor responsible for the change in output.

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